Thinking about Network Routing through Routing Algebra

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Based on
"Network Routing with Path Vector Protocols: Theory and Applications"
João Luís Sobrinho, SIGCOMM'03

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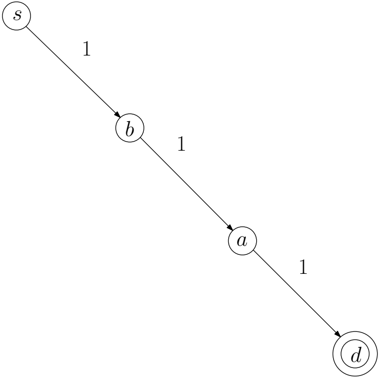
About Me

- First year PhD Student, with
  - **Offer** from Boon
  - **Official Reject** from Penn Committee
  - Very **low** TOEFL and GRE Scores
- Received **Bachelor** from Shanghai Jiaotong University, China
- Half year **Internship** in HP Labs China
- Machine Learning, Data Management and analysis
- ACM **Honored** Class
- Yang Li $\rightarrow$ Young Lee

Path Vector Protocol for Shortest Path

Given destination d, for every node

1. Receive routing message from neighbor
2. Check whether an update required
3. Advertise its new route if updated
4. Repeat until convergence



Compared with Other Metrics

Metric	Usage
Sum	Shortest Path
Min	Capacity, bandwidth
Multiply on [0,1]	Reliability
Combination of Sum and Min	Both shortest path and capacity
??	Customer-Provider and peer-peer

Two Questions

For those Metrics

1. Would Path Vector Protocol be able to generate **Correct Answer**?
2. Is it guaranteed that the Protocol would **Converge** after a period of time?

BGP

- Belongs to this class
  - export
  - PVT
  - import
- Local Preference
- Algebra framework

Customer-Provider and Peer-Peer relationship

- Policy-based routing
- Three kinds of links
  - **Customer Links**

    Providers $\rightarrow$ Customers
  - **Provider Links**

    Customers $\rightarrow$ Providers
  - **Peer Links**

    Peers $\rightarrow$ Peers
- Preferences
  - Customer Path is good
  - Both Peer Path and Provider Path are less good

IGRP, Interior Gateway Protocol

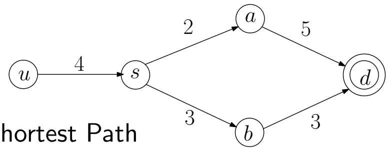
- Composite Metric
  - Distance
  - Bandwidth
- Cost function

  $$f((d,b)) = d + \frac{k}{b}$$
- Special Algorithm
- More in "Maximizable routing metrics". Mohamed Gouda and Marco Schneider. ICNP 1998 and ToN 11(4), 2003.

Represent a Network through Algebra

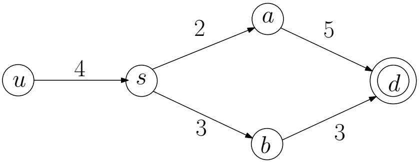
Seven-Tuple

$(W, \preceq, L, \Sigma, \phi, \oplus, f)$



	Meaning	In Shortest Path
W	Weights Value Space	$\mathbf{R}^+$
$\preceq$	Weights Order	$\leq$
L	Links	$\{l(s,a); l(a,d); ...\}$
Σ	Path	$\{s(sad); s(bd); ...\}$
ϕ	Least Preferred Path	$\{s(sd); ...\}$
$\oplus$	Path Extension $L \times \Sigma \to \Sigma$	$l(s,a) \oplus s(ad) = s(sad)$
f	Weight function $\Sigma \to W$	$f(s(sad)) = 7$

Properties



Maximality	$\forall_{\alpha \in \Sigma - \{\phi\}}, f(\alpha) \prec f(\phi)$
Absorption	$\forall_{l \in L}, l \oplus \phi = \phi$
Monotonicity	$\forall_{l \in L} \forall_{\alpha \in \Sigma}, f(\alpha) \preceq f(l \oplus \alpha)$
Strict Monotonicity	$\forall_{l \in L} \forall_{\alpha \in \Sigma - \{\phi\}}, f(\alpha) \prec f(l \oplus \alpha)$
Isotonicity	$\forall_{l \in L} \forall_{\alpha, \beta \in \Sigma}$, $f(\alpha) \preceq f(\beta) \Rightarrow f(l \oplus \alpha) \preceq f(l \oplus \beta)$

Isotonicity, e.g

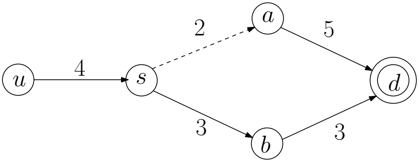
IGRP

- Both Shortest Path and Capacity
- Link Extension

  (d_1, b_1) combines (d_2, b_2) yields $(d_1 + d_2, \min(b_1, b_2))$
- Weight Function

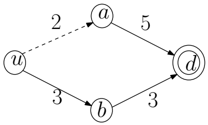
  $f((d, b)) = d + \frac{k}{b}$
- Now we have
  - $f((2, k)) = 3 < 5 = f((1, \frac{k}{4}))$
  - Combined to $(1, \frac{k}{4})$, they becomes $(3, \frac{k}{4})$, $(2, \frac{k}{4})$
  - $f((3, \frac{k}{4})) = 7 > 6 = f((2, \frac{k}{4}))$

Optimal-paths In-tree

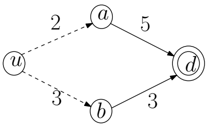


- Based on specific destination d
- A Tree
- Two conditions
  - Node u belongs to the in-tree ⇒ the only path in the in-tree from u to d is an optimal path.
  - Node u does not belong to the in-tree ⇒ no optimal path from u to d.

Local-optimal-paths In-tree



T_d

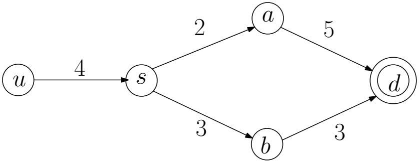


$V_u(T_d)$

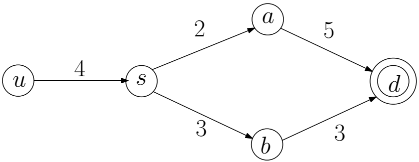
- Based on specific destination d
- V is a set of paths originating in an out-neighbor of node u, for destination d. {s(ad);s(bd)}
- $\overline{V}$ is the extension of the paths to node u. {s(uad);s(ubd)}
- Two conditions for **local-optimal path in-tree** T_d
  - Node u belongs to the in-tree ⇒ the only path in the in-tree from u to d is an **local-optimal path**.
  - Node u does not belong to the in-tree ⇒ no **local-optimal path** from u to d.

Optimal Path with Optimal Subpaths

Proposition 1. *If the algebra is* **isotone** *as well as* **monotone**, *then there is an* **optimal path** *from node u to node d such that all of the* **subpaths** *with destination at d are* **optimal paths** *on their own.*



Intuitive Explanation for Prop. 1



Subpaths

Monotonicity cycle with Negative weight

Isotonicity Assume, Optimal Path is s(usad), and s(sbd) is better than s(sad), why not choose s(usbd)?

Example IGRP

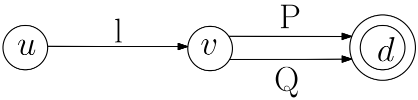
Guarantee the Optimality

Proposition 2. *Given the monotone algebra, every* **local-optimal-paths in-tree** *is an* **optimal-paths in-tree** *if and only if the algebra is* **isotone***.*

"$\Leftarrow$" part

Four cases(op for optimal-path in-tree, lop for local-optimal-path in-tree)

- u doesn't belongs to any of them
- u belongs to op but not lop, consider the out-neighbor of u on the optimal path
- u belongs to lop but not op, the local-optimal path is already a optimal path
- u belongs to both, similar to Prop. 1



"$\Rightarrow$" part

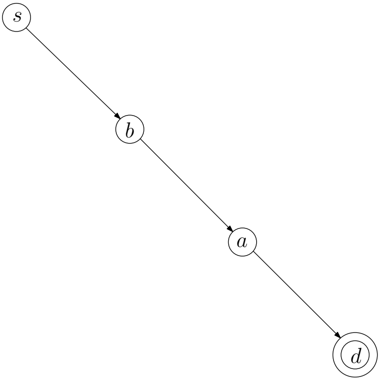
Consider the example on the paper

- $f(P) \preceq f(Q)$ but $f(l \oplus P) \succ f(l \oplus Q)$
- Optimal-path in-tree will contains P
- Local-optimal-path in-tree w.r.t. u will contains Q

Again, Path Vector Protocol

Let's review our Path Vector Protocol, with algebra

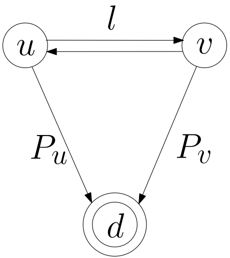
1. Receive a Path P with signature s
2. Try $\oplus$ to get more paths, use f to check if better
3. Advertise its new route if updated
4. Repeat until convergence



Necessity of Monotonicity

- $f(P_u) = f(P_v)$
  $f(P_u) \succ f(l \oplus P_u)$
  $f(P_v) \succ f(l \oplus P_v)$
- Synchronized as following

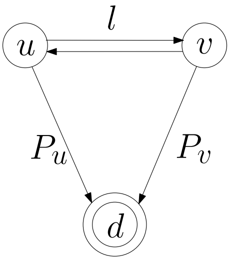
u	v
P_u	P_v
$l \oplus P_v$	$l \oplus P_u$
P_u	P_v
$\dots$	$\dots$



What about equality?

- $f(P_u) = f(P_v)$
  $f(P_u) = f(l \oplus P_u)$
  $f(P_v) = f(l \oplus P_v)$
- it is still possible that

u	v
P_u	P_v
$l \oplus P_v$	$l \oplus P_u$
P_u	P_v
$\dots$	$\dots$



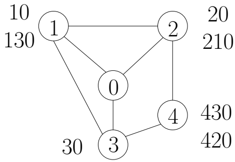
Minimal Number of Links Preferences

Proposition 4'. *If the algebra is* **strict monotone** *then whatever the network, the path vector protocol* **converges**.

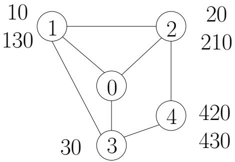
Proposition 4. *If the algebra is monotone and nodes prefer* **paths with minimum number of links** *among those with the same weight, then whatever the network, the path vector protocol* **converges**.

Remark. We do not like fickle people.

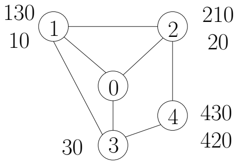
Related to Stable Path Problem



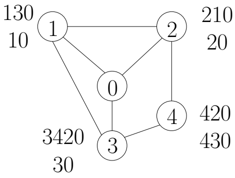
Shortest 1



Shortest 2



Good Gadget



Bad Gadget

Relative Path Preferences

Proposition 5. *The algebra is* **monotone** *if and only if* **there are** *relative path preferences for paths with the same weight that guarantee convergence of the path vector protocol in every network.*

Remark.A result given by combining the Necessity of Monotonicity and Prop.4.

Freeness

Definition.

$$\forall_{cycle\{u_n \ldots u_1 u_0\}} \forall_{w \in W - \{f(\phi)\}} \exists_{0 < i \leq n} \forall_{\alpha \in \Sigma}$$

$$f(\alpha) = w \Rightarrow f(l(u_i, u_{i-1}) \oplus \alpha) \neq w$$

Proposition 3. *If the algebra is* **monotone** *and the network is* **free***, then whatever the relative preference given to paths with the same weight, the path vector protocol* **converges***.*

Remark. Preferring path with minimum hops $\Rightarrow$ Strict Monotonicity $\Rightarrow$ Freeness

Customer-Provider Peer-Peer relationships

- Links types
  - **Customer Links, c**

    Providers → Customers
  - **Provider Links, p**

    Customers → Providers
  - **Peer Links, r**

    Peers → Peers
- $L = \{c, r, p\}$,$\Sigma = L \cup \{\epsilon, \phi\}$,$W = 0, 1, 2, +\infty$

		signature			
	$\oplus$	ϵ	c	r	p
	c	c	c	ϕ	ϕ
label	r	r	r	ϕ	ϕ
	p	p	p	p	p

For example, $c \oplus r = \phi$ means a peer path cannot be extended to become a customer path.

- f is given by

$$f(\epsilon) = 0$$
$$f(c) = 1$$
$$f(r) = f(p) = 2$$
$$f(\phi) = +\infty$$

- Both monotone and isotone

Backup Paths

- Two components Link, (avoidance level, link type), (y, c)
  - Primary paths have avoidance level 0
  - Backup paths have different avoidance level
- Algebra
  - $L = (R^+ \times \{c, r\}) \cup \{p\}$
  - $\Sigma = (R_0^+ \times \{c, r, p\}) \cup \{\epsilon, \phi\}$
  - $W = (R_0^+ \times \{1, 2\}) \cup \{0, +\infty\}$

$\oplus$	ϵ	(x,c)	(x,r)	(x,p)
(y,c)	$(0,c)$	(x,c)	$(x+y,c)$	ϕ
(y,r)	$(0,r)$	(x,r)	$(x+y,r)$	$(x+y,r)$
p	$(0,p)$	(x,p)	(x,p)	(x,p)

For example, $(y,c) \oplus (x,p) = \phi$ means a node does not export a path learned from one provider to a different provider.
$(y,c) \oplus (x,r) = (x+y,c)$,means that a customer can export a peer path to one of its providers, but increase the avoidance level.

- f is given

$$f(\epsilon) = 0,\ f((x,c)) = (x,1)$$
$$f((x,r)) = f((x,p)) = (x,2),\ f(\phi) = +\infty$$

- Monotone but not isotone

Thanks!